



1920M

2019-2020 MATHCOUNTS Exam

Friday, December 6, 2019

INSTRUCTIONS

1. DO NOT BEGIN THIS EXAM UNTIL YOUR PROCTOR TELLS YOU.
2. This is a thirty question SHORT ANSWER test. All answers must be recorded in the correct location on the separate answer sheet.
3. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer. Ties will be broken for top placement positions based on the highest numbered question answered correctly. If students are still tied, the process is repeated for the remainder of questions in reverse order. Exact ties will be broken at the sole discretion of the Math Club chair.
4. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
5. Figures are not necessarily drawn to scale.
6. Units are not necessary unless the question asks for time, where AM or PM should be specified.
7. Give all answers in simplest form, rationalizing the denominator if necessary. If you get a fractional answer, express it as a common fraction unless otherwise indicated. If the answer is dealing with money, then round to the nearest hundredth.
8. Please make sure to write your name where indicated.
9. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to finish your exam.
10. When you finish the exam, please go over your answers again to check your work.

Questions for this exam were authored by Ananya Jain, Joseph Kaim, Conor Kennedy, Evan Kim.

ANSWER SHEET

Name
Grade

Score 1	Score 2	Final
Initial 1	Initial 2	

Do not write in shaded regions.

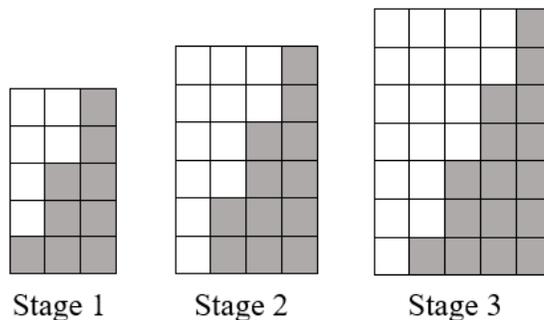
	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
1-15 Total			

	Answer	1 or 0	1 or 0
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
16-30 Total			

- John and Johnna each roll an icosahedron. What is the probability that John rolls a prime and Johnna rolls a multiple of 3?
- Billie has a triangle with side lengths of 3. How much more area would she be adding to her triangle if she increased the side lengths by 1?
- New York is approximately 215 miles from Boston. If Cassius can average 50 miles an hour while going from New York to Boston, and 80 miles an hour back, what will his average speed be over the entire journey? Answer to the nearest tenth.
- Convert 14641_7 to base 10.
- If a sphere has a radius of $2\sqrt{2}$ inches, what is the ratio of its volume to its surface area? Answer in simplest form.
- Four people are arguing about the time they are going to sleep. Stated are their conditions:
 - Person 1 will go to bed only immediately after person 3.
 - Person 2 will only sleep first if person 4 goes last.

Given that Person 4 sleeps third, which order do they sleep in? Give your answer as a 4-digit number with the first person to sleep as the first digit, the second person to sleep as the second digit and so on.

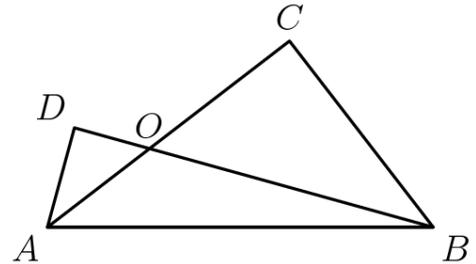
- A set of six positive integers has a mean and median of 11. If the unique mode of this dataset is 4, what is its maximum possible range?
- At Redmond Middle School, there are 50 eighth graders. Of them, 29 play the flute and 31 play the recorder (they're different instruments), with all of those not playing the flute playing the recorder. At the same time, 10 are learning to play guitar, with all guitar players playing the flute and not the recorder. How many flute players play only the flute?
- If the pattern shown below continues, how many shaded squares would be in the 6th stage?



- In how many ways can you pick three cards from a standard 52-card deck (order does not matter) such that at least two of the cards are black and exactly one face card (Jack, Queen, or King) is chosen?

11. Mr. Keanu and Mr. Reeves mow a lawn together. Mr. Keanu can mow a single lawn in 45 minutes and Mr. Reeves can mow a lawn in 30 minutes. If Mr. Keanu starts mowing at 2:00 PM and Mr. Reeves begins mowing at 2:15 PM, what time do they finish mowing the lawn?
12. A tower is made by stacking cylinders, each subsequent cylinder having half the diameter and half the height of the one under it. If the very bottom cylinder has a diameter of 4 and a height of 4, what is the total volume?

13. Consider the figure to the right. Given that $\angle D = \angle C = 90^\circ$, $\angle DOA = 60^\circ$, $AD = \sqrt{3}$, and $BC = 2\sqrt{3}$, Find DC.



14. Spider Rick has 3 legs and needs to pick 3 socks from his drawer. On a normal day with his sock drawer full, he has 6 blue socks and 8 red socks. If he requires that at least 2 of his socks match, what is the probability that the first 3 socks he selects meets his requirements as a percentage?
15. Let $\triangle ABC$ have sides of length 7, 8, and 9. If $\triangle DEF$ is similar to $\triangle ABC$, has integer side lengths and has two sides of length 42 and 54, then what is the absolute difference between the perimeters of the two triangles?
16. What is the remainder of 7^{2019} when divided by 100?
17. Triangle ABC is inscribed in a circle. If $\angle BAC = 44^\circ$ and the measure of $\angle ABC$ is three times the measure of $\angle BCA$, what is the measure of minor arc AB ?
18. An equilateral triangle is inscribed inside a circle, which is then inscribed inside a regular hexagon. If the area of the triangle is $4\sqrt{3}$, what is the area of the region outside the triangle but inside the hexagon?
19. How many times a day do all the digits of a 12-hour digital clock form an arithmetic sequence? Note that the clock does not use a leading zero (it displays 1:23, not 01:23).
20. Jack and Jill are celebrating their shared birthday by having a cake. They decide to cut the cake in an interesting way. First, they split the cake into thirds, then they split one of the thirds into fifths and another of the thirds into quarters. Given that Jack had exactly half of the cake but fewer slices than Jill, how many slices did Jill have?

21. If the sum of the roots of a quadratic equation is 5, and the product of the roots is $\frac{3}{2}$, what is the quadratic equation in form $ax^2 + bx + c$?
22. Take a triangle $\triangle ABC$ with circumcenter O . Construct point D such that $\overline{AO} \perp \overline{AD}$ and $\angle DCA = \angle BCA$. If $BC = 8$ and $DC = 10$, find AC if B does not lie on segment \overline{DC} .
23. After flipping six coins, I choose one of the coins at random. If this coin shows tails, I flip it to heads. Then, I choose a second coin at random (could be the same as the first coin). If this coin currently shows heads, I change it to tails. Now, what is the expected number of tails showing?
24. What is the sum of the x -coordinates of all intersection points shared by $f(x) = 2x^2 + 3x + 1$ and $g(x) = 3x^2 + 5x - 3$?
25. The imperial unit system was first defined in 1824 and spread throughout the world by the British empire. It has since largely been replaced by the metric system for international exchange. Within the imperial system there are many different lengths that do not necessarily equate to integral measurements of other units in the same system, so you know things get complicated when it comes to converting to metric, which is why the following conversion approximations have been supplied. Given that a fathom is 2.02667 yards, an inch is 2.5 centimeters, there are 1,012.69 fathoms per nautical mile, and a nautical mile is 368.249 rods, how many yards are there in 6 meters?
26. Consider the set of x values that suit the equation $y < 20x^2 + 60x + 5$ where $y = 0$. Given that x can satisfy the equation if it is from $-\infty$ to a , or b to ∞ , with b being greater than a , what is $a + b$?
27. Consider a right triangle $\triangle ABC$ with $\angle B = 90^\circ$, $AB = 6$, and $BC = 8$. A line ℓ is constructed perpendicular to the hypotenuse at C . Another line through the midpoint of the hypotenuse, M , and B is extended to intersect line ℓ at P . Find AP .
28. Biff and Jiff stand on opposite sides of a net. Every second they each either pick up a sponge and throw it over to the other side or fail to pick up a sponge with equal probability. If Biff starts with 3 sponges and Jiff starts with 6, what is the probability that Biff gets rid of all his sponges first?
29. A right triangle is contained by lines $y = -2x + 8$, $y = -x + 9$, and $y = \frac{x}{2} + b$, where b is a real number. The longer leg of the triangle has length $3\sqrt{5}$. Find its area.
30. Let $N = 3^{-3/2}$. If $N^x = 9\sqrt{3}$, find x .