RMSMC

Redmond Middle School Math Club

2122M

2021-2022 MATHCOUNTS Exam

Wednesday, January 12, 2022

INSTRUCTIONS

1. DO NOT BEGIN THIS EXAM UNTIL YOUR PROCTOR TELLS YOU.
2. This is a thirty question SHORT ANSWER test. All answers must be recorded in the correct location on the separate answer sheet.
3. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer. Ties will be broken for top placement positions based on the highest numbered question answered correctly. If students are still tied, the process is repeated for the remainder of questions in reverse order. Exact ties will be broken at the sole discretion of the Math Club chair.
4. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
5. Figures are not necessarily drawn to scale.
6. Units are not necessary unless the question asks for time, where AM or PM should be specified.
7. Give all answers in simplest form, rationalizing the denominator if necessary. If you get a fractional answer, express it as a common fraction unless otherwise indicated. If the answer is dealing with money, then round to the nearest hundredth.
8. Please make sure to write your name where indicated.
9. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to finish your exam.
10. When you finish the exam, please go over your answers again to check your work.

Questions for this exam were authored by Nishita Bhakar, Joseph Kaim, Conor Kennedy, Evan Kim, and Sarah Wen.

ANSWER SHEET

Name	
Grade	

Score 1	Score 2	Final
Initial 1	Initial 2	

Do not write in shaded regions.

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
1-15 Total			

	Answer	1 or 0	1 or 0
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
16-30 Total			

1. Consider a weighted coin with 60% probability of landing on heads. What is the probability it will land tails at least twice in three tosses?

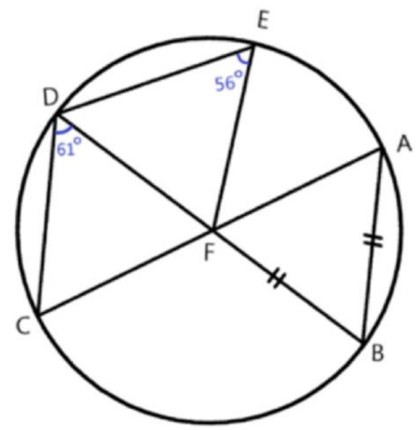
2. A circle is inscribed in a square. It has a radius of $\frac{3\pi}{7}$. What is the ratio of the circle's area to the area of the square?

3. What is the area of a triangle with angles of $30°$, $60°$, and $90°$, and a hypotenuse of 24?

4. Find the difference between the smallest and largest possible values of $M\ M\ 6$ if different letters can represent the same digit.

$$\begin{array}{r} R\ M\ S \\ \underline{+\ R\ H\ S} \\ M\ M\ 6 \end{array}$$

5. Liam has been locked in the Mathematical Arcs Association headquarters due to breaking the Law of the Arkangels. To get out, he must correctly answer a question about arcs and angles. He is given the following information and diagram:

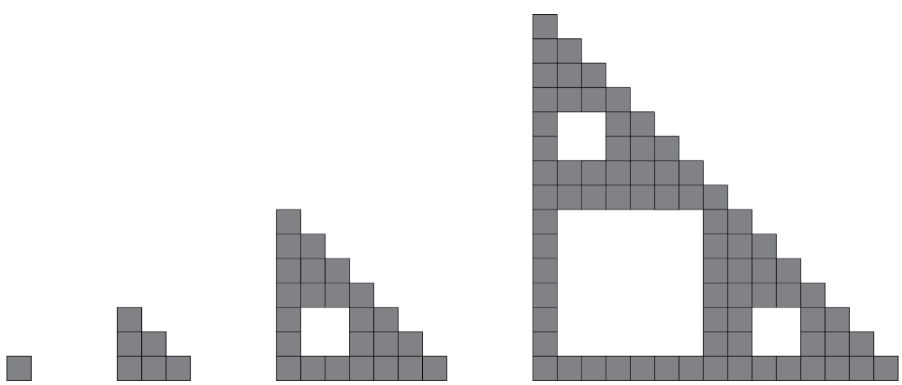


$\angle CDF = 61°$
$\angle DEF = 56°$
$\overline{AB} = \overline{BF}$
The ratio of $\angle EFA$ to $\angle DFE$ is 3:4.

What is $\angle DFE$?

6. What are the last two digits of 12^{16}?

7. Clark is laying out gray tiles on the floor as shown below:



Stage 1 Stage 2 Stage 3 Stage 4

How many gray tiles will there be after the 6th stage?

8. How many base-10 divisors does 110101101_2 have?

9. Ryan has a special set of weights. Instead of their weights only adding when placed on a scale, some also multiply the total weight currently on the scale. For example, if the weight on the scale was 2, and a $3 \times$ weight is added, the weight changes to 6. He plays with 3 of his weights, A, B, and C by placing them on a scale in various orders. The results are as follows:

   A then B = 26

   A then C then B = 65

   C then B then A = 41

   All 3 weights are integers with either an addition or multiplication operation associated with them. What would the result be if he placed A then B then C?

10. As part of a challenge, Jon is throwing a ball up in the air and wants his friend Andy, who is standing on the roof of a building, to catch it. The curve the ball follows can be modelled by the equation $y = -0.5t^2 + 5.5t + 3$, where t is the time in seconds since Jon threw the ball, and y is the height of the ball in feet. If the building is 15 feet tall, and Andy can only catch the ball when it is above the roof of the building, for how many seconds is Andy able to catch the ball?

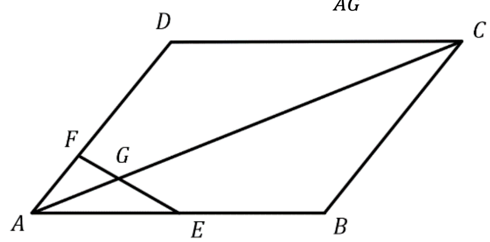
11. Jenny found an extra can of camouflage paint in her garage, and she wants to mess with her friends by painting their backpacks with it. Each backpack is shaped like a half-cylinder (with a semicircle as the base), and the backpacks are 12 inches tall with a radius of 8 inches. If Jenny wants to paint every external side, what is the total area of paint she needs for one backpack (in square inches)? Express your answer in the form $a + b\pi$, where a and b are integers.

12. Andy, Beth, and Charlie are participating as a team in a candy-eating competition. There are a total of 30 candies. Andy and Beth are both allergic to nuts, so they agree that Charlie must eat all 6 candies containing them (but he can still have other candies, too). Eight of the other candies are Jolly Ranchers, which Beth loves, so she eats half of them and equally splits the rest for Andy and Charlie (again, they can eat other candies too). Finally, Andy eats four times as many of the remaining candies as Beth. If Charlie ends up eating five fewer candies in total than Andy, how many candies did Andy eat in total?

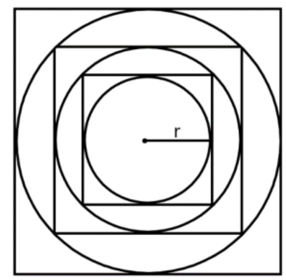
13. The Byers family has 4 daughters and 3 sons, how many ways can they be seated in a row of 7 chairs such that at least 2 daughters are next to each other?

14. In Mathitopia, 1 manth is 5 daios, 1 daio is 10 houras, 1 houra is 80 minos, and 1 mino is 40 secands. Right now it is (manth/daio/houra/mino/secand) 1/4/9/20/5. After 44040 secands have passed, what is the day/time in daio/houra/mino/secand format?

15. In parallelogram $ABCD$, point E is the midpoint of AB, and point F is on AD so that $\frac{AF}{AD} = \frac{1}{3}$. Let G be the point of intersection of AC and FE. Find $\frac{AC}{AG}$.



16. The figure shows a circle circumscribed by a square, which is circumscribed by another circle circumscribed by a square, which is finally circumscribed by a circle and circumscribed by a square. The most inner circle has a radius of 4 centimeters. What is the perimeter of the outside (largest) square in meters?



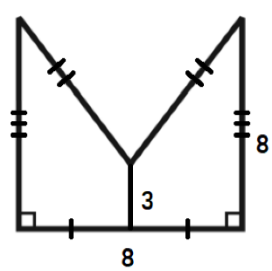
17. A circle is represented by an equation in the form $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) are the coordinates of the center of the circle and r is the radius of the circle. A circle with equation of $(x-3)^2 + (y-7)^2 = 25$ intersects with a line $y = -0.5x + 11$ at two points. What is the product of the y-coordinates?

18. Express $2.1\overline{905}$ in simplest fractional form.

19. $x = 6 + \frac{66}{6+\frac{66}{6+\frac{66}{6+\ddots}}}$. The value x can be expressed in the form $a + b\sqrt{c}$, where a, b, and c are positive integers and c is not divisible by the square of any prime. Find $a + b + c$.

20. What is the sum of the infinite series that begins $5 + 4 + 1 + 2 + \frac{1}{5} + 1 + \frac{1}{25} + \frac{1}{2}$?

21. What is the volume of the solid created when this figure is revolved around its left side?



22. A polynomial $f(x) = 3x^3 + kx + 24$ for some unknown k has three real roots. If two of the roots have a sum of 6, find the value of k.

23. As part of another challenge, Jon is throwing balls into boxes. Each box is worth a certain number of points, and every time Jon throws a ball into the box, he earns that number of points (or loses, if it's negative). The closest box to Jon is worth 1 point, the next is worth -10, the one after is worth 100, then -1000, and so on, alternating positive and negative all the way up to 1 million. If Jon has 8 balls, and he doesn't miss any shots, what is the number of different total scores he is able to obtain?

24. Melina is mixing red and blue paint to create purple paint. She randomly pours anywhere between 0-5 gallons or red paint and 0-5 gallons of blue paint into her mixing bucket (not necessarily a whole number amount). She considers her paint an *ideal purple* if the ratio of red to blue paint by volume is between 0.8 and 0.125 inclusive. What is the probability that the paint she mixes is an *ideal purple*? Assume that she is equally likely to pour any amount of paint between 0 and 5 gallons for each.

25. A marathon is a race of 26.2 miles. Forrest decides that he wanted to run in the 2022 Boston Marathon on April 18. In order to qualify, he must first complete a standard marathon course certified by a national governing body affiliated with the World Athletics and complete with a time of under 3 hours, as he is currently 24 years old. As he only has just over 3 months to prepare, he decides to start running. His current mile time is 10 minutes, far too slow for the race. He decides to look for a coach and finds one by the name of Jackson "Runfast" Mickelson. He claims that he could lower his mile time by five minutes if he practiced for the next three months for five hours a day. He says that Forrest would only start off at about 20 miles a day but would quickly be able to move near 45 miles, just enough to qualify for the Boston Marathon. If he ran a qualifying marathon in New York on April 2 in 2:57 and then ran the Boston Marathon on April 18 in 2:50, how many miles will he have run in the combined marathons, assuming that they are both standard length marathons? Answer as a decimal.

26. Every second a frog at coordinates (x, y) jumps to $(x+3, y+7)$ or $(x+7, y+3)$ with equal probability. The frog starts at the origin, $(0,0)$. What is the probability that after 4 seconds, the frog is less than 30 units from its starting point?

27. Jack and Jill are from mars where a *follar* is worth 6 *vents*. They are building a pile of coins together. At each turn, one of them adds either a 2-vent coin or a 4-vent coin with equal probability (they decide with a coin flip). The first person to place a coin bringing the total value to a whole follar amount (i.e. a multiple of 6 vents) wins. If Jack goes first, what is the probability that he wins?

28. Evaluate $\log_4 8$.

29. The vertices of a triangle lie at $(4, 6)$, $(0, 5)$, and $(-13, -2)$. The centroid of this particular triangle has coordinates of (a, b). What is the product of a and b?

30. $2f(x) + 3f\left(\frac{2x+29}{x-2}\right) = 100x + 80$ for all $x \neq 2$. Find $f(3)$.