RMSMC
Redmond Middle School Math Club

2324M
2023-2024 MATHCOUNTS Exam

Friday, January 12, 2024

INSTRUCTIONS

1. DO NOT BEGIN THIS EXAM UNTIL YOUR PROCTOR TELLS YOU.
2. This is a thirty question SHORT ANSWER test. All answers must be recorded in the correct location on the separate answer sheet.
3. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer. Ties will be broken for top placement positions based on the highest numbered question answered correctly. If students are still tied, the process is repeated for the remainder of questions in reverse order. Exact ties will be broken at the sole discretion of the Math Club chair.
4. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
5. Figures are not necessarily drawn to scale.
6. Units are not necessary unless the question asks for time, where AM or PM should be specified.
7. Give all answers in simplest form, rationalizing the denominator if necessary. If you get a fractional answer, express it as a common fraction unless otherwise indicated. If the answer is dealing with money, then round to the nearest hundredth.
8. Please make sure to write your name where indicated.
9. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to finish your exam.
10. When you finish the exam, please go over your answers again to check your work.

Questions for this exam were authored by Layla Kaim, Emma Li, and Sarah Wen.

ANSWER SHEET

Name			Score 1	Score 2	Final
Grade			Initial 1	Initial 2	

Do not write in shaded regions.

	Answer	1 or 0	1 or 0
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
1-15 Total			

|	Answer	1 or 0	1 or 0
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
16-30 Total			

1. If Sam wrote the integers from 1 to 120 on a blackboard, how many even digits did he write?

2. Given that 2 *fearless* equal 9 *evermore*,3 *evermore* equals 4 *red*,and 8 *red* equal 1 *debut*, how many *fearless* are in 3 *debut*?

3. A rhombus has a perimeter of 40 centimeters and a diagonal of 16 centimeters. What is the area of the rhombus, in square centimeters?

4. What is $\binom{9}{2} + \binom{8}{3} + \binom{7}{4}$?

5. How many digits are in the base 5 representation of the base 10 number 2024?

6. Compute $\frac{2+4+6+\cdots\ldots+2022+2024}{3+6+9+\cdots\ldots+3033+3036}$

7. Find the maximum possible rational root of $f(x) = x^2 + kx + 24$ if k is an integer.

8. Two similar triangles have corresponding sides in the ratio of 3:5. If the area of the smaller triangle is 27 square units, what is the area of the larger triangle in square units?

9. Triangle ABC has coordinate points at A(3, -2), B(7, 4), C(-5, 10). The triangle is translated 3 units down and 4 units to the left, followed by a reflection over the line y = x. What is the sum of all 6 coordinate values of Triangle ABC after such transformations?

10. Yesterday was Wednesday. What day of the week will it be 345 days from tomorrow?

11. Given $a : b : c = 2 : 1 : 4$, the value of $\frac{3a+2b}{4c-a}$ can be written as a fraction of the form $\frac{x}{y}$. What is x+y?

12. A chord of Circle A is 16 centimeters long and 6 centimeters away from the center of Circle A. What is the diameter of Circle A?

13. Abigail wanted to make 12 butterscotch cupcakes with 4 cups of butterscotch, 1 cup of vegetable oil, 3 eggs, and 4 cups of flour. To make 16 butterscotch cupcakes, she would need X eggs, but to make 15 butterscotch cupcakes, she would need Y cups of flour. What is X * Y?

14. In a regular pentagon ABCDE, line segments AD and BD are drawn creating 3 triangular sections. What is the measure of angle CBD?

15. A train leaves City A at 9:00 AM traveling towards City B at 60 miles per hour. Another train leaves City B at 10:00 AM traveling towards City A at 80 miles per hour. The distance between City A and City B is 250 miles. The amount of hours between the time the train from City B departs and the time both trains pass each other can be represented as a simplified fraction in form a/b. What is a + b?

16. The largest four-digit integer whose digits are distinct and form a decreasing geometric sequence can be written as ABCD. What is A+B+C+D?

17. Solve the following system of equations:
$$8x + y - z = 46$$
$$3x + 4y - 2z = 27$$
$$4x - y + z = 14$$

Express your answer as (x, y, z).

18. If, in the following diagram, medians CE and AD intersect at P, DP is 4, PE is 3, and DE is 5, then what is the area of AEDC?
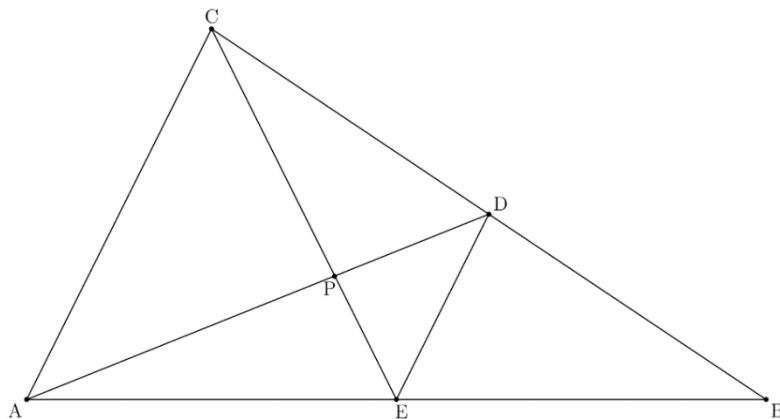
19. What is the y value of the y-intercept of the line that passes through the points (2, 3) and (15, 55)?

20. Find the value of a, such that all real b will satisfy $2ab - 14a + 5b - 35 = 0$.

21. What is $(\log_2 125)/(\log_2 25)$?

22. Jack is playing a game with a standard deck of cards. He will draw cards at random, and earn points based on the value of the card (with A – 1 pt, J – 11 pts, Q – 12 pts, K – 13 pts). However, if he draws the Jack of Hearts, he will get 100 points. What is the expected number of points Jack will earn after one draw? Express your answer as a mixed number.

23. A set of 40 students took a survey that asked if they liked chocolate, strawberry, or vanilla ice cream. The results showed that 23 like chocolate, 21 like strawberry, 16 like vanilla, 7 like both vanilla and strawberry, 11 like both strawberry and chocolate, 9 like chocolate and vanilla, and 3 like all three flavors. What is the probability, as a decimal, a randomly selected student said they didn't like any of the 3 flavors?

24. What is the least amount of time, in minutes, that can pass after 8:00 where the minute hand and the hour hand form an acute angle?

25. What is the product of the 2 infinite series $\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right)\left(3 + \frac{1}{3} + \frac{1}{27} + \frac{1}{243} + \cdots\right)$?

26. A bag contains brown marbles, black marbles, purple marbles, and blue marbles. There are an equal number of brown marbles and black marbles, and five times as many purple marbles as blue marbles. There is a 35% chance of selecting a brown marble first. What is the fewest possible number of purple marbles in the bag?

27. If $\overline{0.144}$ is written as a fraction $\frac{a}{b}$ with a and b being positive integers with a greatest common divisor of 1, what is $a * b$?

28. A cone of radius 4 and height 12 is split into a fulcrum and proportional cone. The smaller cone has a volume of π and the volume of the fulcrum can be written as $AB\pi$, where A represents the tens place and B represents the ones place. What is $A - B$?

29. If $f(x) = x^3 - 8x^2 + 9x - 18$ has roots a, b, and c, then find $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac}$.

30. Mr. Smith is wrapping presents for his children for Christmas. Unfortunately, he realized that he had almost run out of wrapping paper! If he only has 180 square feet of wrapping paper left right now, what is the largest volume of a box with integer side lengths that he can still cover fully with wrapping paper?